

Decay of light scalar mesons into vector-photon and into pseudoscalar mesons

Francesco Giacosa^(a) and Giuseppe Pagliara^(b)

^(a) *Institut für Theoretische Physik, JW Goethe-Universität,
Max von Laue-Str. 1 D-60438 Frankfurt, Germany*

^(b) *Institut für Theoretische Physik, Ruprecht-Karls-Universität,
Philosophenweg 16, D-69120 Heidelberg, Germany*

May 22, 2009

Abstract

The decays of a light scalar meson into a vector meson and a photon ($S \rightarrow V\gamma$) are evaluated in the tetraquark and quarkonium assignments of the scalar states. A link with the radiative decays $\phi \rightarrow S\gamma$ is established: experimental results for $S \rightarrow V\gamma$ will allow to understand if the direct, quark-loop contribution to $\phi \rightarrow S\gamma$ or the kaon-loop contribution is dominant. Also strong decays $S \rightarrow PP$ -where P denotes a pseudoscalar meson- are investigated: the tetraquark assignment works better than the quarkonium one. It is then also discussed why the tetraquark assignment is favoured with respect to a loosely bound kaonic molecular interpretation of $a_0(980)$ and $f_0(980)$ mesons.

1 Introduction

Theoretical studies of the reactions $S \rightarrow V\gamma$, where $S = f_0 \equiv f_0(980)$, $a_0 \equiv a_0(980)$ and $V = \rho, \omega$, have been performed recently [1, 2, 3, 4, 5, 6]. Experimental effort is on-going at Wasa@Cosy to determine the corresponding decay widths [7].

The aim of this work is to calculate $S \rightarrow V\gamma$ in the tetraquark ($\bar{q}^2 q^2$) and quarkonium ($\bar{q}q$) pictures of the light scalar mesons (see Refs. [8, 9, 10] and Refs. therein) and to establish a link between the already performed and the future experimental work at KLOE [11, 12] and the on-going analysis at COSY, where complementary processes involving the f_0 and a_0 mesons are studied.

In this paper the evaluation of $S \rightarrow V\gamma$ is performed in two distinct ways, which we briefly describe in the following:

In way (1) we use previous results [13] obtained by studying the processes, precisely measured by the KLOE collaboration, $\phi \rightarrow f_0(980)\gamma \rightarrow \pi^0\pi^0\gamma$ and $\phi \rightarrow a_0(980)\gamma \rightarrow \pi^0\eta\gamma$. The analysis of [13] was based on the assumption that the direct (i.e. quark-loop driven) decay mechanism of Fig. 1.a (via derivative coupling of scalar-to-pseudoscalar mesons) dominates, see the next section for details. The results for $S \rightarrow V\gamma$ turn out to be large compared to other works [1, 2, 3, 4, 5, 6] and seemingly unlikely (especially in the $\bar{q}q$ assignment). In particular, the decay $\rho \rightarrow \sigma\gamma$ turns out to be sizably larger than the corresponding experimental value [14, 15]. Also, using (Sakurai's version of) Vector Meson Dominance (VMD), the $\gamma\gamma$ decays turn out to be 2 (3) orders of magnitudes larger than the experimental results, in the tetraquark (quarkonium) scenario. For all these reasons, the scenario in which the mechanism in Fig. 1.a dominates the radiative ϕ decay is disfavored. However, its final rejection (or, eventually, resurrection) will be possible as soon as the experimental results for $S \rightarrow V\gamma$ will be known.

In way (2) we start from $\gamma\gamma$ data of f_0 and a_0 reported in Ref. [14]. Using VMD, the decays $S \rightarrow V\gamma$ are evaluated: smaller results are obtained, in line with other studies and with the experimental value $\rho \rightarrow \sigma\gamma$, both in the tetraquark and the quarkonium scenarios. These results for $S \rightarrow V\gamma$, if confirmed, unequivocally show that the mechanism in Fig. 1.a is negligible when studying ϕ decays, in turn meaning that the kaon-loop approach of Fig. 1.b dominates.

Besides radiative decays, an important result of our work comes from the study of decays into two pseudoscalar mesons of the full nonet $\{f_0(600), k(800), f_0(980), a_0(980)\}$, both in the tetraquark and quarkonium pictures by using the decay amplitudes measured in Refs. [16, 17]. Quite remarkably, the tetraquark assignment works better than the quarkonium one: in fact, large decay widths for $f_0(600)$ and $k(800)$ are found, while in the quarkonium assignment small -and thus unphysical- values for the widths are obtained. Moreover, the scalar mixing angle in the $\bar{q}^2 q^2$ assignment is compatible with zero, in agreement with the degeneracy of f_0 and a_0 being $\frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] + [d, s][\bar{d}, \bar{s}])$ and $\frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] - [d, s][\bar{d}, \bar{s}])$ respectively. (A large value of the mixing angle would inevitably spoil the mass degeneracy, which is one of the key features in favour of a tetraquark interpretation of scalars). On the contrary, in the $\bar{q}q$ assignment the mixing angle turns out to have a sign which is opposite to the one expected from basic considerations of the axial anomaly. The paper is organized as it follows: after a brief recall of radiative ϕ decays (Section 2), we turn the attention to the results in the tetraquark (Section 3) and quarkonium scenarios (Section 4) respectively. Then, in section 5 further discussions and conclusions are presented.

2 Recall of $\phi \rightarrow S\gamma$

2.1 General considerations

As a first step we briefly review the mechanisms of the radiative ϕ decays: the reaction $\phi \rightarrow S\gamma$, where $S = f_0, a_0$ is followed by a subsequent decay of S into two pseudoscalar mesons. In the case in which f_0, a_0 are tetraquark or quarkonium states (i.e. correspond to a preformed and preexisting, non-dynamically generated states, see the discussion in Ref. [10]) the chain $\phi \rightarrow S\gamma \rightarrow PP\gamma$ where $P = \pi, \eta$ can occur following the two mechanisms described in Fig. 1: Fig. 1.a corresponds to a direct coupling of the ϕ meson to a photon and S , while in Fig. 1.b ϕ couples to γ and S via a kaon-loop. If f_0, a_0 are loosely bound $\bar{K}K$ molecular states, then only the diagram in Fig. 1.b contributes.

In the case of the f_0 meson, the general interaction Lagrangian which describes both mechanisms is given by:

$$\begin{aligned} \mathcal{L}_{int, f_0} = & c_{\phi f_0 \gamma} \phi_{\mu\nu} f_0 F^{\mu\nu} + c_{f_0 \pi \pi} f_0 (\partial_\mu \vec{\pi})^2 + d_{f_0 \pi \pi} f_0 \vec{\pi}^2 + \\ & c_{f_0 K K} f_0 (\partial_\mu K^+) (\partial^\mu K^-) + d_{f_0 K K} f_0 (K^+ K^-) + \dots \end{aligned} \quad (1)$$

where dots refer to the analogous terms with the neutral kaon states. One has:

(a) The direct coupling $c_{\phi f_0 \gamma} \phi_{\mu\nu} f_0 F^{\mu\nu}$ (where $\phi_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$ with ϕ_μ the field of the ϕ meson, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic strength tensor) induces the ϕ decay, Fig. 1.a. The f_0 meson decays in $\pi^0 \pi^0$ via the parameters $c_{f_0 \pi \pi}$ and $d_{f_0 \pi \pi}$.

(b) The ϕ couples to f_0 via a kaon-loop, see Fig. 1.b. Then, the f_0 subsequently decays. This mechanism, although large- N_c suppressed, has been regarded as dominant by many authors [18].

Similarly in the a_0 case: a coupling $c_{\phi a_0 \gamma} \phi_{\mu\nu} a_0 F^{\mu\nu}$ describes the point-like (quark-loop driven) coupling of the ϕ mesons to a_0 and γ , and analogous coupling constants $c_{a_0 \pi \eta}$, $d_{a_0 \pi \eta}$, $c_{a_0 K K}$, $d_{a_0 K K}$ are introduced. It is important to stress that in the considered tetraquark/quarkonium interpretation of a_0 and f_0 both mechanisms necessarily take place. It is however still unsettled which one, if any, is dominant.

On the contrary, in the molecular assignment only the parameters $d_{f_0 K K}$ and $d_{a_0 K K}$ do not vanish (all other constants can be set to zero, in particular $c_{\phi f_0 \gamma}$). The coupling to ϕ and also the coupling to $\pi\pi$ are driven by kaon loops via a subsequent $\bar{K}K\pi\pi$ and $\bar{K}K\pi\eta$ coupling [5].

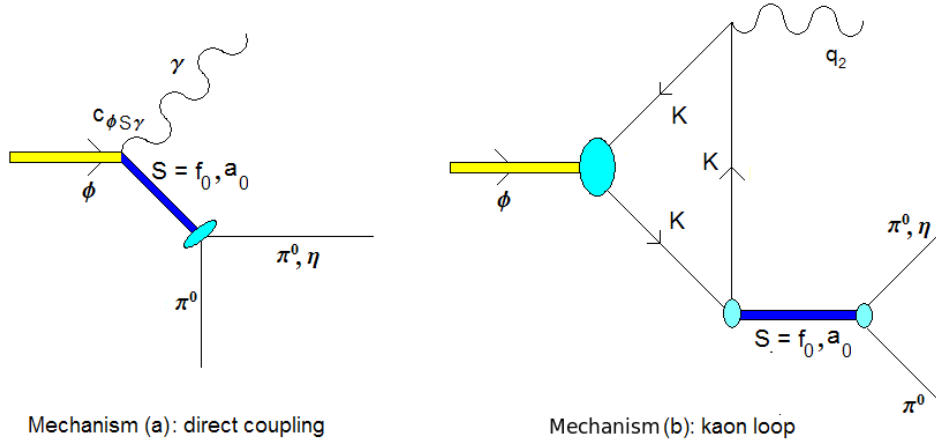


Figure 1: Mechanism (a): direct (quark-loop) contribution. Mechanism (b): kaon-loop contribution.

2.2 Fit to line shapes via Fig. 1.a

In Ref. [13], using the formalism developed in Ref. [19], only the diagram of Fig. 1.a has been taken into account in the limit $d_{f_0\pi\pi} = d_{f_0KK} = d_{a_0\pi\eta} = d_{a_0KK} = 0$, which corresponds to the limit in which *only* derivative couplings of scalar to pseudoscalar mesons are retained. As originally discussed in Ref. [20], derivative interactions are potentially interesting because, due to the extra dependence on the phase space, allow to describe peaked line shapes $\phi \rightarrow f_0(980)\gamma \rightarrow \pi^0\pi^0\gamma$ and $\phi \rightarrow a_0(980)\gamma \rightarrow \pi^0\eta\gamma$, see details in [13]. By fitting the theoretical curves to the experimental data one can determine - among others- the coupling constants $c_{\phi f_0\gamma}$ and $c_{\phi a_0\gamma}$, which parametrize the point-like coupling. A precise determination was not possible because the fits depend only marginally on the couplings to kaons c_{f_0KK} and c_{a_0KK} ; for this reasons fits with different values of the latter have been performed. Nevertheless, a clear and parameter-independent outcome has been obtained:

$$c_{\phi f_0\gamma} \gtrsim 0.25 \text{ GeV}^{-1}, \quad c_{\phi a_0\gamma} \gtrsim 0.25 \text{ GeV}^{-1}. \quad (2)$$

Moreover, when using the strong decay amplitudes obtained in the experimental works of Refs. [16, 17] (see also [21]):

$$\left| A_{f_0\pi\pi}^{\text{exp}} \right| = 2.88 \pm 0.22, \quad \left| A_{f_0KK}^{\text{exp}} \right| = 5.91 \pm 0.77, \quad \left| A_{a_0\pi\eta}^{\text{exp}} \right| = 3.33 \pm 0.15, \quad \left| A_{a_0KK}^{\text{exp}} \right| = 3.59 \pm 0.44, \quad (3)$$

one finds (tiny errors omitted)

$$c_{\phi f_0\gamma} = 0.801 \text{ GeV}^{-1}, \quad c_{\phi a_0\gamma} = 0.385 \text{ GeV}^{-1}. \quad (4)$$

Other choices for the amplitudes are possible by taking the values listed in the compilation of Refs. [22, 23], where a variety of results are summarized. They would lead to slightly different values for the point-like couplings but would not change the qualitative conclusions.

3 Results in the tetraquark assignment

3.1 The Lagrangian

Be $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ with $V_\mu = \text{diag}\{\frac{1}{\sqrt{2}}(\rho^0 + \omega_N), \frac{1}{\sqrt{2}}(-\rho^0 + \omega_N), \phi_S\}$ the diagonal matrix with the neutral vector mesons, where $\omega_N = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ and $\phi_S = \bar{s}s$. The physical states ω and ϕ are

given by

$$\begin{pmatrix} \omega \\ \phi \end{pmatrix} = \begin{pmatrix} \cos(\varphi_V) & \sin(\varphi_V) \\ -\sin(\varphi_V) & \cos(\varphi_V) \end{pmatrix} \begin{pmatrix} \omega_N \\ \phi_S \end{pmatrix}. \quad (5)$$

The mixing angle $\varphi_V = -3.74^\circ$ is close to zero. The quantity $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic strength tensor and $Q = \text{diag}\{2/3, -1/3, -1/3\}$ the charge matrix.

The original discussion of tetraquark states as candidates for the light scalar mesons was presented in Ref. [24] and revisited in Ref. [25]. Here we introduce the tetraquark scalar nonet $\mathcal{S}^{[4q]}$ using the formalism of Refs. [26, 27]:

$$\mathcal{S}^{[4q]} = \frac{1}{2} \begin{pmatrix} [\bar{d}, \bar{s}][d, s] & -[\bar{d}, \bar{s}][u, s] & [\bar{d}, \bar{s}][u, d] \\ -[\bar{u}, \bar{s}][d, s] & [\bar{u}, \bar{s}][u, s] & -[\bar{u}, \bar{s}][u, d] \\ [\bar{u}, \bar{d}][d, s] & -[\bar{u}, \bar{d}][u, s] & [\bar{u}, \bar{d}][u, d] \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{2}}(f_B - a_0^0) & -a_0^+ & k^+ \\ -a_0^- & \sqrt{\frac{1}{2}}(f_B + a_0^0) & -k^0 \\ k^- & -\bar{k}^0 & \sigma_B \end{pmatrix}, \quad (6)$$

where the tetraquark content, in terms of diquark-antidiquark composition, has been made explicit. Note, the commutator $[\cdot, \cdot]$ reminds that the all diquarks are in the antisymmetric antitriplet $\bar{3}_F$ representation. In particular, the states $\sigma_B[4q] = \frac{1}{2}[u, d][\bar{u}, \bar{d}]$ and $f_B[4q] = \frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] + [d, s][\bar{d}, \bar{s}])$ refer to bare (unmixed) tetraquark scalar-isoscalar states. The physical states σ and $f_0 \equiv f_0(980)$ are then given by

$$\begin{pmatrix} \sigma \\ f_0(980) \end{pmatrix} = \begin{pmatrix} \cos(\varphi_S) & \sin(\varphi_S) \\ -\sin(\varphi_S) & \cos(\varphi_S) \end{pmatrix} \begin{pmatrix} \sigma_B \\ f_B \end{pmatrix}. \quad (7)$$

where φ_S is the scalar mixing angle.

The pseudoscalar nonet is described, as usual, by the matrix

$$\mathcal{P} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{2}}\eta_N & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{2}}\eta_N & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \quad (8)$$

where $\eta_N \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ and $\eta_S \equiv \bar{s}s$. The physical fields arise as $\eta = \eta_N \cos \varphi_P + \eta_S \sin \varphi_P$, $\eta' = -\eta_N \sin \varphi_P + \eta_S \cos \varphi_P$. The value $\varphi_P = -36.0^\circ$ [28] is used in the following for definiteness; it lies roughly in the middle of the phenomenological range from -32° to -44° found in various studies (variation within this range does not imply any qualitative change in the following).

The interaction Lagrangian involves γVS couplings (parametrized by b_1 and b_2) and the (derivative) scalar-pseudoscalar SPP couplings (parametrized by c_1 and c_2):

$$\begin{aligned} \mathcal{L}_{q^2 q^2} &= b_1 \mathcal{S}_{ij}^{[4q]} \text{Tr} [Y^j V_{\mu\nu} Y^i Q] F^{\mu\nu} - b_2 \mathcal{S}_{ij}^{[4q]} \text{Tr} [Y^j Y^i V_{\mu\nu} Q] F^{\mu\nu} + \\ &c_1 \mathcal{S}_{ij}^{[4q]} \text{Tr} [Y^j (\partial_\mu \mathcal{P}^t) Y^i (\partial^\mu \mathcal{P})] - c_2 \mathcal{S}_{ij}^{[4q]} \text{Tr} [Y^j Y^i (\partial_\mu \mathcal{P}) (\partial^\mu \mathcal{P})] \end{aligned} \quad (9)$$

where $(Y^i)_{jk} = \varepsilon_{ijk}$. Note, the Lagrangian of Eq. (1) with $d_{f_0\pi\pi} = d_{f_0KK} = 0$ is part of Eq. (9).

The trace structure is such that the terms b_1, b_2 (c_1, c_2) describe the $V\gamma$ (PP) decays according to Fig. 2.a and 2.b respectively. In Fig. 2.a a rearrangement and a subsequent decay of the tetraquark state take place, while in Fig. 2.b one has first a quark-antiquark annihilation into one (or more) gluon(s) and then the decay. Note, although the mechanism of Fig. 2.b is suppressed by a factor N_c w.r.t. to Fig. 2.a, it has an important role in phenomenology as discussed in Ref. [26].

3.2 Strong decays

The strong decays are parametrized by c_1 (Fig. 2.a, fall apart decay) and c_2 (Fig 2.b). We recall that, in virtue of the derivatives, the (tree-level) decay widths read:

$$\Gamma_{S \rightarrow P_1 P_2} = \frac{k}{8\pi M_S^2} |A_{SP_1 P_2}|^2, \text{ with amplitude } A_{SP_1 P_2} = c_{SP_1 P_2} \frac{(M_S^2 - M_{P_1}^2 - M_{P_2}^2)}{2}, \quad (10)$$

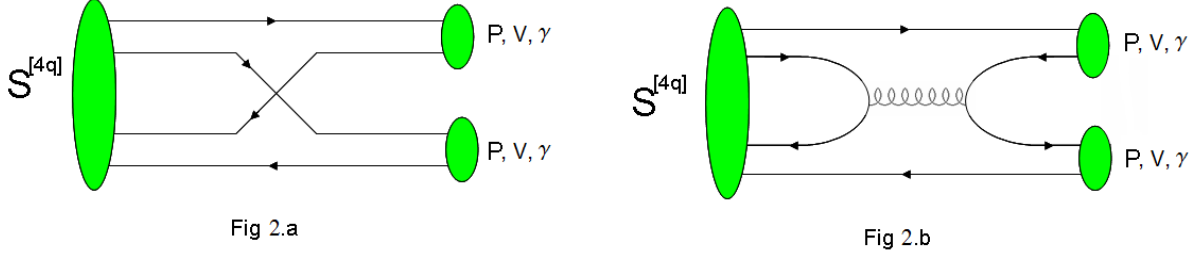


Figure 2: Tetraquark into two pseudoscalars (PP), vector and photon ($V\gamma$), or two photons ($\gamma\gamma$) via quark rearrangement (Fig. 2.a) and annihilation in (at least) one gluon (Fig. 2.b).

where k is the three-momentum of an outgoing particle. The coefficients $c_{SP_1P_2}$ are obtained by evaluating the traces of Eq. (9). Some relevant ones are reported in Table 1. Most notably, the term proportional to c_2 (Fig. 2.b) has two effects: it sizably increases $f_B \rightarrow \bar{K}K$ (thus allowing for a correct description of the enhanced $f_B \rightarrow \bar{K}K$ amplitude in comparison to $a_0 \rightarrow \bar{K}K$) and it is responsible for a nonzero $f_B \rightarrow \pi\pi$ amplitude. (Note, in Ref. [29] an instanton-induced term is responsible for a non-zero $f_B \rightarrow \pi\pi$ amplitude; here such an instanton term is not needed because the mechanism of Fig. 2.b is capable of a proper description of phenomenology. Future studies are required to understand which possibility is realized).

Table 1. Relevant decay coupling constants

$S \rightarrow P_1 P_2$	$c_{S \rightarrow P_1 P_2}$
$f_B \rightarrow \bar{K}K$	$\sqrt{2} \left(\sqrt{2}c_1 + \frac{3}{\sqrt{2}}c_2 \right)$
$f_B \rightarrow \pi\pi$	$\sqrt{\frac{3}{2}} \cdot (\sqrt{2}c_2)$
$a_0 \rightarrow \bar{K}K$	$\sqrt{2} \cdot \left(\sqrt{2}c_1 + \frac{1}{\sqrt{2}}c_2 \right)$
$a_0 \rightarrow \pi\eta$	$-2c_1 \sin(\varphi_P) + \sqrt{2}c_2 \cos(\varphi_P)$
$k \rightarrow \pi K$	$\sqrt{3} \cdot \left(\sqrt{2}c_1 + \frac{1}{\sqrt{2}}c_2 \right)$
$\sigma_B \rightarrow \pi\pi$	$\sqrt{\frac{3}{2}} \cdot (2c_1 + 2c_2)$

We perform a fit with 3 parameters c_1 , c_2 and φ_S to the four experimental values of Eqs. (3). The minimum is found for

$$c_1 = 5.19 \pm 1.78 [\text{GeV}^{-1}], \quad c_2 = 3.84 \pm 1.80 [\text{GeV}^{-1}], \quad \varphi_S = 1.2^\circ \pm 8.0^\circ; \quad \chi^2 = 1.17. \quad (11)$$

The small value $\chi^2 = 1.17$ implies that all the amplitudes can be well reproduced. The scalar mixing angle φ_S is small and compatible with zero. This is an important fact: a large scalar mixing angle would spoil the experimentally well measured degeneracy of f_0 and a_0 .

As a result of Eq. (11) one has $\Gamma_{\sigma \rightarrow \pi\pi} = 379 \pm 52 \text{ MeV}$, $\Gamma_{k \rightarrow K\pi} = 330 \pm 82 \text{ MeV}$ for masses $M_\sigma = 0.6 \text{ GeV}$ and $M_k = 0.9 \text{ GeV}$. While the errors refer to the parameters c_1 , c_2 and φ_S only (and are thus underestimated), it is quite remarkable that large decays of σ and k can be obtained from parameters which were fitted in the f_0 and a_0 sector *only*. One obtains a qualitative acceptable description of the full scalar nonet below 1 GeV. This is not the case in the quark-antiquark assignment, see next sections.

3.3 Radiative decays

We first recall that the tree-level decay rates read

$$\Gamma_{S \rightarrow V\gamma} = \frac{(M_S^2 - M_V^2)^3}{8\pi M_S^3} g_{SV\gamma}^2, \quad \Gamma_{V \rightarrow S\gamma} = \frac{(M_V^2 - M_S^2)^3}{24\pi M_V^3} g_{SV\gamma}^2. \quad (12)$$

The couplings $g_{SV\gamma}$ are functions of the two coefficients b_1 and b_2 entering in Eq. (9). They are reported in Table 2 where, for transparency, the limit $\varphi_V = 0$ and $\varphi_S = 0$ is considered, which corresponds to the amplitudes of the bare fields ω_N , ϕ_S and σ_B , f_B . Being the mixing angles φ_V and φ_S small, these amplitudes are already close to their real values. A simple use of the relations of Eqs. (5)-(7) would allow to obtain the amplitude for the physical states. For completeness we report also the ϕ decay channels, but we stress that, also when assuming that Fig. 1.a is dominant, the tree-level decay rates cannot be used due to the closeness to threshold and a full study of the line shapes, as for instance the one in Ref. [13], should be performed.

Independently on the precise value of the parameters, the tetraquark scenario makes the following predictions: $\Gamma_{a_0 \rightarrow \omega\gamma} \simeq \Gamma_{f_0 \rightarrow \rho\gamma}$ and $\Gamma_{a_0 \rightarrow \rho\gamma} \simeq \Gamma_{f_0 \rightarrow \omega\gamma}$. Also, the mechanism of Fig. 2.a strongly enhances the (quark-loop driven) decay mode $\phi \rightarrow f_0\gamma$.

Table 2. $SV\gamma$ decay coupling constants in the $\bar{q}^2 q^2$ case

ϕ_S	$g_{SV\gamma}$	ω_N	$g_{SV\gamma}$	ρ	$g_{SV\gamma}$
$\phi_B \rightarrow a_0\gamma$	$\frac{b_1}{\sqrt{2}}$	$a_0 \rightarrow \omega_N\gamma$	$\frac{b_2}{2}$	$a_0 \rightarrow \rho\gamma$	$\frac{2b_1+b_2}{6}$
$\phi_S \rightarrow f_B\gamma$	$\frac{b_1+2b_2}{3\sqrt{2}}$	$f_B \rightarrow \omega_N\gamma$	$\frac{2b_1+b_2}{6}$	$f_B \rightarrow \rho\gamma$	$\frac{b_2}{2}$
$\phi_S \rightarrow \sigma_B\gamma$	0	$\omega_N \rightarrow \sigma_B\gamma$	$\frac{-b_1+b_2}{3\sqrt{2}}$	$\rho \rightarrow \sigma_B\gamma$	$\frac{b_1+b_2}{\sqrt{2}}$

We now calculate the radiative decays in the two ways mentioned in the Introduction: first, by using the results of Ref. [13], which assume a dominance of the diagram of Fig. 1.a in radiative ϕ decays, and then starting from $\gamma\gamma$ data of f_0 and a_0 . As we shall see, different results are obtained.

Way 1: We fix $\varphi_S = 1.2^\circ$ as determined by the fit to strong decays. Using Eq. (4) we determine $b_1 = 0.61 \text{ GeV}^{-1}$ and $b_2 = 1.46 \text{ GeV}^{-1}$. Note, the ratio $b_2/b_1 \sim 2$ is already problematic because it implies a dominance of the large- N_c suppressed contribution of Fig. 2.b. The results for $S \rightarrow V\gamma$ decays are summarized in Table 3.

Table 3: $S \rightarrow V\gamma$ in the $\bar{q}^2 q^2$ case using Ref. [13].

Mode	Decay [keV]	Mode	Decay [keV]
$a_0 \rightarrow \rho\gamma$	535	$a_0 \rightarrow \omega\gamma$	1406
$f_0 \rightarrow \rho\gamma$	1005	$f_0 \rightarrow \omega\gamma$	463
$\rho \rightarrow \sigma\gamma$	876	$\omega \rightarrow \sigma\gamma$	20

Moreover: $\Gamma_{\phi \rightarrow \sigma\gamma} = 0.06 \text{ keV}$ (small).

The values in Table 3 are large when compared to the results obtained via meson-loop contributions, which are typically smaller than 20 keV [1, 3, 5]. The main point is that large decay widths such as those in Table 3 are possible only if the direct mechanism of Fig. 1.a dominates.

Both decay modes $\omega \rightarrow \sigma\gamma$ and $\rho \rightarrow \sigma\gamma$ turn out to be larger than the present experimental knowledge ($\Gamma_{\rho \rightarrow \sigma\gamma} \simeq 6 \text{ keV}$ and $\Gamma_{\omega \rightarrow \sigma\gamma} \leq 6 \text{ keV}$, see Ref. [14, 15]). While this represents an argument against this scenario, one should not forget that the theoretical expressions for these decays strongly depend on the σ mass and on finite-width corrections.

Vector meson dominance can be introduced in the model by applying the shift $V_{\mu\nu} \rightarrow V_{\mu\nu} + \frac{\sqrt{2}\alpha}{g_\rho} Q F_{\mu\nu}$, where $g_\rho = 6.1$ and $\alpha \simeq 1/137$ is the fine structure constant. Although VMD cannot be

used for precise calculations of on-shell decays of photons (see discussion in Ref. [30]), it is a valuable phenomenological tool to estimate the order of magnitude. The results are:

$$\Gamma_{f_0 \rightarrow \gamma\gamma} \simeq 72 \text{ keV}, \Gamma_{a_0 \rightarrow \gamma\gamma} \simeq 40 \text{ keV}, \Gamma_{\sigma \rightarrow \gamma\gamma} \simeq 26 \text{ keV}. \quad (13)$$

These results are sizably larger than the experimental results [14]:

$$\Gamma_{f_0 \rightarrow \gamma\gamma}^{\text{exp}} = 0.29_{-0.9}^{+0.7} \text{ keV}, \Gamma_{a_0 \rightarrow \gamma\gamma}^{\text{exp}} = 0.3 \pm 0.1 \text{ keV}, \Gamma_{\sigma \rightarrow \gamma\gamma}^{\text{exp}} = 0.5\text{-}4 \text{ keV}. \quad (14)$$

Thus, the present experimental evidence is against these solutions and thus to the hypothesis that mechanism of Fig. 1.a plays a dominant role in the ϕ decay.

In principle we could also have used the results of the non-structure model of Ref. [31], in which Fig. 1.a is also regarded as dominant, but non-derivative interactions of scalar to pseudoscalar mesons are used. The coupling constants $c_{\phi f_0 \gamma}$ and $c_{\phi a_0 \gamma}$ correspond indeed to the lower limit of Eq. (2). Smaller results (for instance $\Gamma_{a_0 \rightarrow \omega\gamma} \simeq 77 \text{ keV}$) follow, but the strong amplitudes $A_{SP_1 P_2}$ (fixed by fitting to KLOE line shapes) determined in Ref. [31] turn out to be significantly smaller than Eq. (3). As a consequence, in this scheme the σ and k mesons would have a width of $\sim 100 \text{ MeV}$ (or even smaller), which is in clear disagreement with the data.

Way 2: We first obtain $b_1 = 0.075 \text{ GeV}^{-1}$ and $b_2 = 0.083 \text{ GeV}^{-1}$ from the $\gamma\gamma$ decays $\Gamma_{f_0 \rightarrow \gamma\gamma}^{\text{exp}} = 0.29_{-0.9}^{+0.7} \text{ keV}$, $\Gamma_{a_0 \rightarrow \gamma\gamma}^{\text{exp}} = 0.30 \pm 0.1 \text{ keV}$ [14] (under the assumptions that they are dominated by the direct, quark-loop in the tetraquark assignment). Note, in this case $b_2/b_1 \simeq 1$ in line with the ratio c_2/c_1 in the strong sector. As a first consequence a small $\Gamma_{\sigma \rightarrow \gamma\gamma} = 0.14 \text{ keV}$ (which corresponds to the direct, quark-loop decays and neglects pion loops) is found. Then, via VMD, we evaluate the $S \rightarrow V\gamma$ decay rates, which are summarized in Table 4.

Table 4: $S \rightarrow V\gamma$ in the $\bar{q}^2 q^2$ case using VMD.

Mode	Decay [keV]	Mode	Decay [keV]
$a_0 \rightarrow \rho\gamma$	4.0	$a_0 \rightarrow \omega\gamma$	4.9
$f_0 \rightarrow \rho\gamma$	3.1	$f_0 \rightarrow \omega\gamma$	3.4
$\rho \rightarrow \sigma\gamma$	5.1	$\omega \rightarrow \sigma\gamma$	0.003

Moreover: $\Gamma_{\phi \rightarrow \sigma\gamma} = 0.004 \text{ keV}$ (very small).

In this case the predicted $\rho \rightarrow \sigma\gamma$ and $\omega \rightarrow \sigma\gamma$ are in agreement with the experiment [15]. As a further important consequence, the couplings $c_{\phi f_0 \gamma} = 0.050 \text{ GeV}^{-1}$ and $c_{\phi a_0 \gamma} = 0.054 \text{ GeV}^{-1}$ are determined. They are factor of 10 *smaller* than the values of Eq. (4). It is evident that the contribution of Fig. 1.a is reduced of a factor of 100 and is negligible in this scenario. The only possibility is that the kaon loop of Fig. 1.b dominates the radiative decay of the ϕ mesons.

4 Results in the quarkonium assignment

4.1 The Lagrangian

A quark-antiquark interpretation of the light scalars is disfavored by the mass pattern, large N_c and various phenomenological arguments (see Refs. [9, 32] and Refs. therein). However, being not yet fully ruled out, it is instructive to perform the study within this assignment. The quark-antiquark nonet is encoded in the matrix

$$S[\bar{q}q] = \begin{pmatrix} \bar{u}u & \bar{d}u & \bar{s}u \\ \bar{u}d & \bar{d}d & \bar{s}d \\ \bar{u}s & \bar{d}s & \bar{s}s \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{2}}\sigma_B & a_0^+ & k^+ \\ a_0^- & -\frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{2}}\sigma_B & k^0 \\ k^- & \bar{k}^0 & f_B \end{pmatrix}, \quad (15)$$

where the quark content has been made explicit. The scalar-isoscalar states read $\sigma_B = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ and $f_B = \bar{s}s$. The physical states σ and $f_0 \equiv f_0(980)$ arise via mixing:

$$\begin{pmatrix} \sigma \\ f_0(980) \end{pmatrix} = \begin{pmatrix} \cos(\varphi_S) & \sin(\varphi_S) \\ -\sin(\varphi_S) & \cos(\varphi_S) \end{pmatrix} \begin{pmatrix} \sigma_B \\ f_B \end{pmatrix}. \quad (16)$$

The Lagrangian for the full-nonet of quark-antiquark, including the non-derivative flavor symmetry braking term, reads [33, 34]:

$$\mathcal{L}_{\bar{q}q} = c_d \text{Tr} \left[\mathcal{S}^{[\bar{q}q]} \partial_\mu \mathcal{P} \partial^\mu \mathcal{P} \right] - c_m \frac{B}{2} \text{Tr} [2\mathcal{S}^{[\bar{q}q]} \mathcal{P} M \mathcal{P} + \mathcal{S}^{[\bar{q}q]} \mathcal{P} \mathcal{P} M + \mathcal{S}^{[\bar{q}q]} M \mathcal{P} \mathcal{P}] + b \text{Tr} \left[\mathcal{S}^{[\bar{q}q]} V Q \right] F^{\mu\nu} \quad (17)$$

where $M = \text{diag}\{m_u, m_d, m_s\}$ is the diagonal matrix of current quark masses and the parameter B is related to the pion and kaon masses as $M_\pi^2 = 2Bm_u$ and $M_K^2 = B(m_u + m_s)$.

4.2 Strong decays

We proceed as in the tetraquark case by performing a fit of the three constants c_d , c_m and φ_S to the four experimental amplitudes of Eq. (3). The theoretical expressions for all the decays can be found in Ref. [34]. Only one acceptable solution is found:

$$c_d = 8.72 \pm 0.44 \text{ GeV}^{-1}, \quad c_m = 6.01 \pm 1.56 \text{ GeV}^{-1}, \quad \varphi_S = -23.7^\circ \pm 2.3^\circ; \quad \chi^2 = 0.02. \quad (18)$$

Although the χ^2 is very small, and thus the amplitudes of Eq. (3) can be correctly reproduced for the above values of the parameters, one obtains as a consequence that $\Gamma_{\sigma \rightarrow \pi\pi} = 170 \pm 17 \text{ MeV}$, $\Gamma_{k \rightarrow K\pi} = 218 \pm 25 \text{ MeV}$ ($M_\sigma = 0.6 \text{ GeV}$ and $M_k = 0.9 \text{ GeV}$ have been used). These decay widths are too small when compared to experiments. This is a clear drawback of the quark-antiquark assignment: the parameters obtained from f_0 and a_0 resonances do *not* allow for a description of the broad k and σ states. Note, as a result of the fit $c_m/c_d \sim 0.7$, thus sizable. A fit based only on the chiral symmetric c_d term would provide a large χ^2 .

A second more subtle but also decisive drawback is the following: a negative mixing angle φ_S is a clear outcome of the fit. In the generalized Nambu Jona-Lasinio model, which provides still one of the main reasons in favour of a $\bar{q}q$ interpretation of light scalars, the mixing in the scalar sector is driven by the 't Hooft term which solves the $U_A(1)$ anomaly problem in the pseudoscalar sector. In fact, the 't Hooft term induces the mixing of isoscalar states both in the pseudoscalar and the scalar sectors and it turns that the mixing angles φ_P and φ_S should have the opposite sign [35]. Being φ_P negative, a positive φ_S is expected for a quark-antiquark nonet (see also the discussion in Ref. [34] and Refs. therein). Note, the same conclusion has been obtained in Ref. [36] where a generalized σ model with an anomaly term is studied. Thus, the fact that our fit provides a negative φ_S represents a further argument against a quarkonium interpretation of light scalar mesons. Indeed, a positive φ_S is the outcome of Refs. [34], where the scalar quarkonium nonet is placed above 1 GeV.

4.3 Radiative decays

The theoretical amplitudes for $S \rightarrow V\gamma$ decays in the $\bar{q}q$ case are reported in Table 5, where φ_S is kept free but $\varphi_V = 0$. Independently from the way (1) or (2) described later on, the following ratios are obtained: $\Gamma_{a_0 \rightarrow \omega\gamma} / \Gamma_{a_0 \rightarrow \rho\gamma} \simeq \Gamma_{f_0 \rightarrow \rho\gamma} / \Gamma_{f_0 \rightarrow \omega\gamma} \simeq \Gamma_{\rho \rightarrow \sigma\gamma} / \Gamma_{\omega \rightarrow \sigma\gamma} \simeq 9$.

Table 5: $SV\gamma$ decay coupling constants in the $\bar{q}q$ case

ϕ_S	$g_{SV\gamma}$	ω_N	$g_{SV\gamma}$	ρ	$g_{SV\gamma}$
$\phi_S \rightarrow a_0\gamma$	0	$a_0 \rightarrow \omega_N\gamma$	$\frac{b}{2}$	$a_0 \rightarrow \rho\gamma$	$\frac{b}{6}$
$\phi_S \rightarrow f_0\gamma$	$\frac{b \cos(\varphi_S)}{3}$	$f_0 \rightarrow \omega_N\gamma$	$-\frac{b \sin(\varphi_S)}{6}$	$f_0 \rightarrow \rho\gamma$	$-\frac{b \sin(\varphi_S)}{2}$
$\phi_S \rightarrow \sigma_B\gamma$	$\frac{b \sin(\varphi_S)}{3}$	$\omega_N \rightarrow \sigma\gamma$	$\frac{b \cos(\varphi_S)}{6}$	$\rho \rightarrow \sigma\gamma$	$\frac{b \cos(\varphi_S)}{2}$

Way 1: When assuming the mechanism of Fig. 1.a as dominant, a problem arises: the ratio $|c_{\phi f_0\gamma}/c_{\phi a_0\gamma}| = \sin(\varphi_V)/(2\cos(\varphi_V)\cos(\varphi_S) - \sin(\varphi_V)\sin(\varphi_S)) = 2.1$ (from Eq. (4)) cannot be reproduced for a small value of φ_S . Indeed, a value of φ_S close to $\pm 90^\circ$ is required to explain this ratio: this is in *clear* disagreement with the result of Eq. (18) and implies an unnatural, dominant $\bar{s}s$ content for $f_0(600)$. More in details, the use of Eq. (4) implies $b = -11.8 \text{ GeV}^{-1}$ and $\varphi_S = -99.9^\circ$. As a result, we determine the radiative decays, summarized in Table 6, which turn out to be too large (note the MeV scale!). For instance, the value $\Gamma_{a_0 \rightarrow \omega\gamma} \sim 85 \text{ MeV}$ is clearly unrealistic. Also, $\Gamma_{\rho \rightarrow \sigma\gamma} \sim 410 \text{ keV}$ is incompatible with Ref. [15].

Table 6: $S \rightarrow V\gamma$ in the $\bar{q}q$ case using Ref. [13].

Mode	Decay [MeV]	Mode	Decay [MeV]
$a_0 \rightarrow \rho\gamma$	10.4	$a_0 \rightarrow \omega\gamma$	85.6
$f_0 \rightarrow \rho\gamma$	69.88	$f_0 \rightarrow \omega\gamma$	6.73
$\rho \rightarrow \sigma\gamma$	0.41	$\omega \rightarrow \sigma\gamma$	0.15

By using VMD the $\gamma\gamma$ decays read $\Gamma_{f_0 \rightarrow \gamma\gamma} \simeq 772 \text{ keV}$, $\Gamma_{a_0 \rightarrow \gamma\gamma} \simeq 1767 \text{ keV}$, $\Gamma_{\sigma \rightarrow \gamma\gamma} \simeq 92 \text{ keV}$, which are 3 order of magnitudes larger than the values of Eq. (14). Varying φ_S does not improve the overall situation. There is no need to discuss this scenario any further: a quarkonium scenario together with a dominant Fig. 1.a is surely ruled out.

Way 2: Using $\Gamma_{a_0 \rightarrow \gamma\gamma}^{\text{exp}} = 0.30 \text{ keV}$ and $\Gamma_{f_0 \rightarrow \gamma\gamma}^{\text{exp}} = 0.29 \text{ keV}$ we obtain $b_1 = 0.23 \text{ GeV}^{-1}$ and $\varphi_S = -20^\circ$. In this case the mixing angle is in agreement with the strong fit of Eq. (18). Note, the corresponding quark-antiquark contribution of the σ decay is in this case $\Gamma_{\sigma \rightarrow \gamma\gamma} = 0.13 \text{ keV}$, thus small¹. The $S \rightarrow V\gamma$ results are summarized in Table 7.

Table 7: $S \rightarrow V\gamma$ in the $\bar{q}q$ case using VMD.

Mode	Decay [keV]	Mode	Decay [keV]
$a_0 \rightarrow \rho\gamma$	4.0	$a_0 \rightarrow \omega\gamma$	33
$f_0 \rightarrow \rho\gamma$	3.3	$f_0 \rightarrow \omega\gamma$	0.61
$\rho \rightarrow \sigma\gamma$	4.7	$\omega \rightarrow \sigma\gamma$	0.53

In this case $\rho \rightarrow \sigma\gamma$ and $\omega \rightarrow \sigma\gamma$ are in agreement with the experiment [15]. The order of magnitude is similar to the one of the values in Table 4. The coupling constants $c_{\phi f_0\gamma} = 0.072 \text{ GeV}^{-1}$ and $c_{\phi a_0\gamma} = -0.0075 \text{ GeV}^{-1}$ are determined, thus implying that Fig. 1.a is negligible (compare with Eq. (4)). There is however a drawback: $\Gamma_{\phi \rightarrow \sigma\gamma} = 3.28 \text{ keV}$, which is much larger than the experimental value $\Gamma_{\phi \rightarrow \sigma\gamma}^{\text{exp}} \lesssim 0.6 \text{ keV}$ obtained in Ref. [11], and thus seems to be excluded. Such a large $\Gamma_{\phi \rightarrow \sigma\gamma} = 3.28 \text{ keV}$ would probably produce a much more pronounced σ peak, which is however not present in the line shapes [12].

5 Discussions and Conclusions

In this work we studied $S \rightarrow PP$, $S \rightarrow V\gamma$ and $S \rightarrow \gamma\gamma$ decays and we discussed the connection of the latter with the radiative process $\phi \rightarrow S\gamma$, which is subject of a detailed experimental analysis by the KLOE group.

A general outcome of $S \rightarrow V\gamma$ ratios (Tables 2 and 5), which does not depend on numerical details, is the predictions of the following properties. Tetraquark scenario: $\Gamma_{a_0 \rightarrow \omega\gamma} \simeq \Gamma_{f_0 \rightarrow \rho\gamma}$ and $\Gamma_{a_0 \rightarrow \rho\gamma} \simeq \Gamma_{f_0 \rightarrow \omega\gamma}$. Quarkonium scenario: $\Gamma_{a_0 \rightarrow \omega\gamma}/\Gamma_{a_0 \rightarrow \rho\gamma} \simeq \Gamma_{f_0 \rightarrow \rho\gamma}/\Gamma_{f_0 \rightarrow \omega\gamma} \simeq \Gamma_{\rho \rightarrow \sigma\gamma}/\Gamma_{\omega \rightarrow \sigma\gamma} \simeq 9$.

¹A small quark-loop contribution of $\sigma \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ into $\gamma\gamma$ is also the outcome of Ref. [37]. Namely, the finite dimension -which is a necessary property of a bound state such as the quarkonium one- is responsible for a smaller $\gamma\gamma$ result than what a local calculation -which neglects the finite extension of the σ field- would deliver.

In order to obtain numerical values, two ways have been followed. When assuming Fig 1.a as the dominant process in the radiative ϕ decays (denoted as way 1, which necessarily implies a preformed tetraquark or quarkonium substructure of the f_0/a_0 mesons), the results for $S \rightarrow V\gamma$ are summarized in Tables 3 and 6 in the $\bar{q}^2 q^2$ and $\bar{q}q$ cases respectively. The $\bar{q}q$ case is surely excluded because of the unrealistically large values of the $S \rightarrow V\gamma$ decays (see Table 6). The $\bar{q}^2 q^2$ case is also disfavored in view of too large rates for $\rho, \omega \rightarrow \sigma\gamma$ and (VMD deduced) $f_0, a_0, \sigma \rightarrow \gamma\gamma$, but still not completely ruled out. Experimental results are needed: if however large decays will be found (such as, for instance, $\Gamma_{a_0 \rightarrow \omega\gamma} \gtrsim 50$ keV) one could infer that a compact structure is compulsory. Being the quarkonium interpretation highly problematic, one would be necessarily left with the tetraquark interpretation as the only possible one.

When starting from $\gamma\gamma$ data of f_0 and a_0 (way 2), the results, summarized in Tables 4 and 7 for the $\bar{q}^2 q^2$ and $\bar{q}q$ cases, turn out to be of the same order of the kaon-loop based calculations [1, 3, 5]. The outcome is in agreement with the $\rho, \omega \rightarrow \sigma\gamma$ data, but a full and consistent analysis should then include both direct and meson-loop driven (not considered here) contributions to $S \rightarrow \gamma\gamma$ and $S \rightarrow V\gamma$ decays (the $\bar{q}q$ case is anyway disfavored because of a too large $\phi \rightarrow \sigma\gamma$ branching ratio). However, a model-independent conclusion can be achieved: small $S \rightarrow V\gamma$ radiative decays -if confirmed experimentally- would surely imply that the ϕ decay is dominated by the kaon loop of Fig. 1.b, independently from the nature of the scalar states (in agreement with the discussion presented in Ref. [38]). This, in turn, may allow for a precise determination of the amplitudes in future updates of the KLOE experiment. It is indeed interesting to notice that the present strong amplitudes as determined by fits to the line shapes of Ref. [12] assuming the dominance of Fig. 1.b (in GeV), read:

$$|A_{f_0\pi\pi}^{\text{kl-kloe}}| = 1.71 \pm 0.7, \quad |A_{f_0KK}^{\text{kl-kloe}}| = 5.4 \pm 1.6, \quad |A_{a_0\pi\eta}^{\text{kl-kloe}}| = 2.8 \pm 0.1, \quad |A_{a_0KK}^{\text{kl-kloe}}| = 3.06 \pm 0.06, \quad (19)$$

and are in rough agreement with the independent results of Eq. (3), what indeed constitutes a remarkable fact.

We notice that the amplitudes of Eq. (3) imply that the tree-level decays of the f_0 and a_0 mesons are large:

$$\Gamma_{f_0 \rightarrow \pi\pi}^{\text{tl}} = 161 \pm 25 \text{ MeV}, \quad \Gamma_{a_0 \rightarrow \pi\eta}^{\text{tl}} = 146 \pm 13 \text{ MeV} \quad (20)$$

(which, without inclusion of the $\bar{K}K$ channel, are already larger than the 50-100 MeV *full* widths reported by PDG [14]. However, the PDG widths refer to the peak widths, which are strongly distorted due to the nearby $\bar{K}K$ threshold. Note, the KLOE result for $f_0 \rightarrow \pi\pi$ is about 60 MeV, but has a large uncertainty, while $a_0 \rightarrow \pi\eta$ is 103 ± 10 MeV, which is also sizable.) The very fact that large decay widths of f_0 and a_0 are found is the reason why a -albeit qualitative- consistent description of a full tetraquark nonet $\{\sigma, k, f_0, a_0\}$ below 1 GeV is possible, as we presented in Section 3.2. We recall also that, quite remarkably, such a consistent description is *not* achieved in the $\bar{q}q$ case: too small σ and k widths are found and the scalar mixing angle is in disagreement with general arguments based on the $U_A(1)$ anomaly. While small $S \rightarrow V\gamma$ (with $S = f_0/a_0$) decay widths (as in Table 4) are also in agreement with a loosely bound kaon molecular state, the latter predict smaller decay widths than Eq. (20): $\Gamma_{a_0 \rightarrow \pi\eta} = 10\text{-}60$ MeV and $\Gamma_{f_0 \rightarrow \pi\pi} \simeq 10\text{-}50$ MeV [5, 39, 40, 41]. Indeed, a small width ($\lesssim 50$ MeV) is a typical characteristic of a loosely bound kaonic state, as emphasized in Ref. [41]. These predictions are however in disagreement with Eq. (20). In particular, this criticism holds in the $a_0\pi\eta$ channel where the errors are smaller; in the recent work Ref. [40] a small width $\Gamma_{a_0 \rightarrow \pi\eta}$ of at most 30 MeV is found, which is at odd with both Eq. (3) and (19). In view of this discussion we regard the tetraquark assignment as the most suitable for the description of light scalar states.

Acknowledgments: We thank Paolo Gauzzi and Cesare Bini for useful discussions. F. G. thanks BMBF for financial support, G.P. acknowledges financial support from the Alliance Program of the Helmholtz Association (HA216/EMMI).

References

- [1] Yu. Kalashnikova, A. E. Kudryavtsev, A. V. Nefediev, J. Haidenbauer and C. Hanhart, Phys. Rev. C **73** (2006) 045203 [arXiv:nucl-th/0512028].
- [2] D. Black, M. Harada and J. Schechter, Phys. Rev. Lett. **88** (2002) 181603 [arXiv:hep-ph/0202069].
- [3] R. Escribano, P. Masjuan and J. Nadal, Phys. Lett. B **670** (2008) 27 [arXiv:0806.3007 [hep-ph]].
- [4] S. Ivashyn and A. Y. Korchin, Eur. Phys. J. C **54** (2008) 89 [arXiv:0707.2700 [hep-ph]].
- [5] T. Branz, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D **78** (2008) 114004 [arXiv:0808.0705 [hep-ph]].
- [6] M. K. Volkov, E. A. Kuraev and Yu. M. Bystritskiy, arXiv:0904.2484 [hep-ph].
- [7] M. Buscher, AIP Conf. Proc. **1030** (2008) 40 [arXiv:0804.2452 [hep-ph]].
- [8] C. Amsler and N. A. Tornqvist, Phys. Rept. **389**, 61 (2004).
- [9] E. Klempt and A. Zaitsev, Phys. Rept. **454** (2007) 1 [arXiv:0708.4016 [hep-ph]].
- [10] F. Giacosa, arXiv:0903.4481 [hep-ph].
- [11] A. Aloisio *et al.* [KLOE Collaboration], Phys. Lett. B **536** (2002) 209 [arXiv:hep-ex/0204012].
A. Aloisio *et al.* [KLOE Collaboration], Phys. Lett. B **537** (2002) 21 [arXiv:hep-ex/0204013].
- [12] F. Ambrosino *et al.* [KLOE Collaboration], Eur. Phys. J. C **49** (2007) 473 [arXiv:hep-ex/0609009].
F. Ambrosino *et al.* [KLOE Collaboration], Phys. Lett. B **634** (2006) 148 [arXiv:hep-ex/0511031].
F. Ambrosino *et al.* [KLOE Collaboration], arXiv:0707.4609 [hep-ex].
- [13] F. Giacosa and G. Pagliara, Nucl. Phys. A **812** (2008) 125 [arXiv:0804.1572 [hep-ph]]. For a summary, see: F. Giacosa and G. Pagliara, arXiv:0812.3357 [hep-ph].
- [14] Particle Data Group, C. Amsler *et al.*, Physics Letters **B667**, 1 (2008).
- [15] R. R. Akhmetshin *et al.* [CMD2 Collaborations], Phys. Lett. B **580** (2004) 119 [arXiv:hep-ex/0310012].
- [16] M. Ablikim *et al.* [BES Collaboration], Phys. Lett. B **607** (2005) 243 [arXiv:hep-ex/0411001].
- [17] D. V. Bugg, V. V. Anisovich, A. Sarantsev and B. S. Zou, Phys. Rev. D **50** (1994) 4412.
- [18] N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. B **315** (1989) 465. N. N. Achasov and V. V. Gu-
bin, Phys. Rev. D **56** (1997) 4084 [arXiv:hep-ph/9703367]. N. N. Achasov and A. V. Kiselev,
Phys. Rev. D **73** (2006) 054029 [Erratum-ibid. D **74** (2006) 059902] [arXiv:hep-ph/0512047].
Yu. S. Kalashnikova, A. E. Kudryavtsev, A. V. Nefediev, C. Hanhart and J. Haidenbauer,
Eur. Phys. J. A **24** (2005) 437 [arXiv:hep-ph/0412340]. H. Nagahiro, L. Roca and E. Oset,
arXiv:0802.0455 [hep-ph]. J. A. Oller, Nucl. Phys. A **714** (2003) 161 [arXiv:hep-ph/0205121].
A. Bramon, R. Escribano, J. L. Lucio M., M. Napsuciale and G. Panzeri, Phys. Lett. B **494**
(2000) 221 [arXiv:hep-ph/0008188]. E. Marco, S. Hirenzaki, E. Oset and H. Toki, Phys. Lett. B **470**
(1999) 20 [arXiv:hep-ph/9903217]. J. E. Palomar, L. Roca, E. Oset and M. J. Vicente Vacas, Nucl.
Phys. A **729** (2003) 743 [arXiv:hep-ph/0306249]. V. E. Markushin, Eur. Phys. J. A **8** (2000) 389
[arXiv:hep-ph/0005164]. R. Escribano, Phys. Rev. D **74** (2006) 114020 [arXiv:hep-ph/0606314].
- [19] F. Giacosa and G. Pagliara, Phys. Rev. C **76** (2007) 065204 [arXiv:0707.3594 [hep-ph]].
- [20] D. Black, M. Harada and J. Schechter, Phys. Rev. D **73** (2006) 054017 [arXiv:hep-ph/0601052].

- [21] D. V. Bugg, Eur. Phys. J. C **47** (2006) 57 [arXiv:hep-ph/0603089].
- [22] V. Baru, J. Haidenbauer, C. Hanhart, A. E. Kudryavtsev and U. G. Meissner, Eur. Phys. J. A **23** (2005) 523 [arXiv:nucl-th/0410099].
- [23] D. Black, A. H. Fariborz, F. Sannino and J. Schechter, Phys. Rev. D **59** (1999) 074026 [arXiv:hep-ph/9808415].
- [24] R. L. Jaffe, Phys. Rev. D **15** (1977) 267. R. L. Jaffe, Phys. Rev. D **15** (1977) 281. R. L. Jaffe and F. E. Low, Phys. Rev. D **19**, 2105 (1979).
- [25] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. Lett. **93** (2004) 212002 [arXiv:hep-ph/0407017].
- [26] F. Giacosa, Phys. Rev. D **74** (2006) 014028 [arXiv:hep-ph/0605191].
- [27] F. Giacosa, Phys. Rev. D **75** (2007) 054007. [arXiv:hep-ph/0611388].
- [28] F. Giacosa, arXiv:0712.0186 [hep-ph].
- [29] G. 't Hooft, G. Isidori, L. Maiani, A. D. Polosa and V. Riquer, Phys. Lett. B **662** (2008) 424 [arXiv:0801.2288 [hep-ph]].
- [30] H. B. O'Connell, B. C. Pearce, A. W. Thomas and A. G. Williams, Prog. Part. Nucl. Phys. **39** (1997) 201 [arXiv:hep-ph/9501251].
- [31] G. Isidori, L. Maiani, M. Nicolaci and S. Pacetti, JHEP **0605** (2006) 049 [arXiv:hep-ph/0603241].
- [32] J. R. Pelaez, Phys. Rev. Lett. **92** (2004) 102001. Mod. Phys. Lett. A **19** (2004) 2879. J. A. Oller and E. Oset, Nucl. Phys. A **620** (1997) 438 [Erratum-ibid. A **652** (1999) 407] N. Mathur *et al.*, Phys. Rev. D **76** (2007) 114505.
- [33] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321** (1989) 311.
- [34] F. Giacosa, T. Gutsche, V. E. Lyubovitskij and A. Faessler, Phys. Rev. D **72** (2005) 094006 [arXiv:hep-ph/0509247]. F. Giacosa, T. Gutsche, V. E. Lyubovitskij and A. Faessler, Phys. Lett. B **622** (2005) 277 [arXiv:hep-ph/0504033].
- [35] T. Hatsuda and T. Kunihiro, Phys. Rept. **247** (1994) 221 [arXiv:hep-ph/9401310].
- [36] J. Schaffner-Bielich and J. Randrup, Phys. Rev. C **59** (1999) 3329 [arXiv:nucl-th/9812032].
- [37] F. Giacosa, T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D **77** (2008) 034007 [arXiv:0710.3403 [hep-ph]].
- [38] C. Hanhart, Eur. Phys. J. A **31** (2007) 543 [arXiv:hep-ph/0609136].
- [39] S. Krewald, R. H. Lemmer and F. P. Sassen, Phys. Rev. D **69** (2004) 016003 [arXiv:hep-ph/0307288].
- [40] R. H. Lemmer, arXiv:0902.1739 [hep-ph].
- [41] J. D. Weinstein and N. Isgur, Phys. Rev. Lett. **48** (1982) 659. J. D. Weinstein and N. Isgur, Phys. Rev. D **41** (1990) 2236.